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HEAVY PARTICLE COLLISIONS

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I. INTRODUCTION

The speaker (LS) is on record as believing that occasionally, for all its obvious disadvantages, talks should be given by those who had not been deeply involved, if at all, in the subject to be covered. Possible advantages include great freedom in the choice of topics, the lack of a compulsion to cover every detail of the topics chosen, and an objectivity (if some ignorance) in accreditation. Neither of us had been knowledgeable in two of the three topics we have chosen. To avoid pitfalls in our treatment of the third, asymmetric charge transfer, we have given a completely qualitative discussion of developments in that area.

We have obviously chosen areas we believe to be of particular significance, but having picked three areas, we cannot begin to give the details necessary for a thorough understanding. Our primary purpose is to interest those who have not read the original papers in doing so.

II. NUCLEAR RESONANCES AND THE PROBABILITY OF K-SHELL IONIZATION

If atomic physics has been enjoying a renaissance because of

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new developments within the field, it is also true that there are areas of atomic physics which are so well understood that agreement between theory and experiment exceeds that in any other field of physics. It is for the latter reason that an area of interplay between atomic physics and another branch of physics can be extremely useful in studying that other branch. Thus, for example, relatively long nuclear half-lives, longer than perhaps picoseconds, can be measured directly, while relatively short nuclear half-lives, shorter than perhaps 10^{-21} seconds, can be determined by measuring the associated half-width, but intermediate half-lives can be difficult to measure by either approach. Now the orbital period of a K-shell electron in a nucleus of atomic number Z is of order $(2\pi a_o/Z)/(Ze^2/\hbar)$ or roughly $10^{-16}/Z^2$ seconds. One can then hope to determine intermediate nuclear half-lives if one can find a measurable effect of the nuclear half-life on a K-shell electron. Consider, for example, the scattering through an angle θ in the center of mass frame of a nuclear projectile P and a target nucleus T, with atomic numbers Z_P and Z_T, respectively. The incident relative kinetic energy $E_i = \frac{1}{2}Mv_i^2$, with M the reduced nuclear mass, is assumed to be in the neighborhood of a resonance with a half-life Δt. (We will talk of ionization, but the argument would be the same for excitation.) We let $P(ion) = P_{fi}(E_i,\theta)$ be the ratio of the probability of the ionization of the K-shell electron in the course of the nuclear scattering process to the probability that there is no ionization during the nuclear scattering. Ionization is from the initial (1s) state i, with normalized wave function $\phi_i(\vec{r})$, to the final continuum state with normalized wave function $\phi_f(\vec{r})$. We will find that P(ion) will depend upon Δt. We expect a significant effect if the nuclear width $\Gamma = \hbar/\Delta t$ is at most of the order of the binding energy U_K of the K-shell electron, or, equivalently, if the half-life or time-delay is at least of the order of the orbital period of the K-shell electron.

In section II we use capital letters for nuclear energies, momenta, coordinates (but not velocities), wave functions, and scattering amplitudes, and small letters for the corresponding electronic properties. We have $\vec{K}_i\hbar = M\vec{v}_i$ and $\vec{K}_f\hbar$ for the incident and emergent relative momenta of the nuclei, E_i and E_f for the associated energies, $\vec{R}$ for the P-T separation and V for the P-T interaction, ϵ_i for the initial K-shell energy of the electron (e^-) and ϵ_f for the e^- energy in its final ionized state, and $\vec{r}$ for the T-e^- separation. We introduce ω via $E_i - E_f = \hbar\omega = \epsilon_f - \epsilon_i$. We will study the problem both in the semi-classical approximation (SCA) and, in the context of quantum theory, in the distorted wave Born approximation (DWBA). We now list a number of approximations which we will make in both the SCA and the DWBA. We will later list additional approximations to be made separately in the SCA and DWBA approximations. The approximations which are made more for convenience of discussion than out of dire necessity are indicated by a star. With the approximations we will make, the mathematics is trivial; in the discussion below the most difficult step is the integration of an exponential. The hard

part is of course the justification of the approximations.

i) *The recoil of T can be neglected.

ii) The P-e^- interaction can be treated as a perturbation; a necessary condition for the validity of (ii) is that $Z_p \ll Z_t$.

iii) The e^--e^- interactions in the neutral target atom can be neglected.

We note that the ejected K-shell electrons have a continuous energy spectrum and therefore provide a poor signature for ionization; a good signature is provided by the K-ray radiation or by one of the Auger electron lines which follow the ionization. (Since the K-ray is emitted by an electron whose initial state is any of a number of p-states --- primarily the 2p state --- with equally populated projections of the angular momentum, and whose final state is an s-state, the K-ray radiation will be spherically symmetric.)

A. The Semi-Classical Approximation

iv') The electron will be treated quantum mechanically, but we assume that the motions of both P and T can be described classically. Indeed, in the lab frame, it follows from assumption (i) that T is always at rest.

v') We assume further that P has an impact parameter of zero with respect to T and moves with constant momentum $\hbar\vec{K}_i$ from time $t = -\infty$ to $t = -\frac{1}{2}\Delta t$, with $\vec{R}(t) = \vec{R}_i v_i (t + \frac{1}{2}\Delta t)$ in this interval, that P then collides with T, the two forming a composity system (with $\vec{R}(t) = 0$) for a time interval Δt, that is, until $t = +\frac{1}{2}\Delta t$, and that P then departs with constant momentum $\hbar\vec{K}_f$, with $\vec{R}(t) = \vec{R}_f v_i (t - \frac{1}{2}\Delta t)$. (For inelastic nuclear scattering, one would simply replace v_i by v_f, the outgoing relative velocity, in this last expression.) The angle θ between $\vec{K}_f$ and $\vec{K}_i$ is fixed by the location of the detector.

The electron is subject to a perturbation H' which for the moment we write simply as $H'(\vec{r},\vec{R}(t))$. The amplitude a_{fi} that the electron will be in a final state f at $t = +\infty$ if it was in an initial state i at $t = -\infty$ is then given, in first order time-dependent perturbation theory, by

$$i\hbar a_{fi} = \int_{-\infty}^{\infty} e^{i\omega t} H'_{fi}[\vec{R}(t)]dt \quad , \tag{2.1}$$

where

$$H'_{fi}[\vec{R}(t)] = \int d^3r \phi_f^*(\vec{r}) H'[\vec{r},\vec{R}(t)] \phi_i(\vec{r}) \quad . \tag{2.2}$$

The dependence of a_{fi} on θ and E_i (through the presence of $\vec{R}(t)$) is often suppressed. With the changes of variables $t \to t \pm \frac{1}{2}\Delta t$, we can rewrite Eq. (2.1) as

$$i\hbar a_{fi} = \exp(-\tfrac{1}{2}i\omega\Delta t)[i\omega^{-1}H'_{fi}(0) + I(-\infty,0;\hat{R}_i)] + \exp(\tfrac{1}{2}i\omega\Delta t)[-i\omega^{-1}H'_{fi}(0) + I(0,\infty;\hat{R}_f)] \quad . \tag{2.3}$$

where

$$I(\alpha,\beta;\hat{R}) \equiv \int_\alpha^\beta \exp(i\omega t) H'_{fi}(\hat{R}v_i t) dt \quad . \tag{2.4}$$

We then have

$$P(\text{ion}) \equiv P_{fi}(E_i,\theta) = |a_{fi}(E_i,\theta)|^2 \quad . \tag{2.5}$$

To proceed further, we would use

$$H'[\vec{r},\vec{R}(t)] = -Z_p e^2/|\vec{r}-\vec{R}(t)| \quad . \tag{2.6}$$

Since P(ion) depends upon a_{fi} which in turn depends upon Δt (through the interference of the two terms in Eq. (2.3) for a_{fi}), a comparison of a theoretical estimate and experimental determination of P(ion) gives an estimate of Δt.

B. A Quantum Approach, in the DWBA

In addition to assuming the validity of the DWBA, we make the following assumptions.

iv") *The nuclear scattering process is an elastic one.

v") *The question of the P-T interaction V did not explicitly arise in the SCA. In the DWBA we need not specify V but we will assume that it is spin-independent and spherically symmetric, that is, that $V = V(R)$. The effects of V(R) are contained in the exact elastic nuclear scattering amplitudes which will appear; these will be denoted in general by $F(\hat{R} \to \hat{K})$, with $\hat{R}$ denoting an arbitrary

emergent direction, and by $P(\hat{K}_f \to \hat{K}_i) \equiv P_{el}(E,\theta)$ for the scattering process of interest.

vi") θ, the angle of scattering of P, is not small.

vii") The dimension over which the effects of V are significant is very small compared to the dimension a_0/Z_T of the K-shell. This is eminently reasonable with regard to the nuclear component of V, and it is not unreasonable even with regard to the Coulomb component of V, since large-angle nuclear scattering is determined largely by V(R) at small R.

viii") We often neglect the difference between K_i and K_f (but never when either appears in an exponent).

Note that, as opposed to the SCA, the DWBA preserves conservation of linear and angular momentum and of energy.

We now invoke the DWBA, as discussed in Taylor (1972), for example, to write for the ionization amplitude $f(ion) \equiv f_{fi}(E_i,\theta)$

$$f(ion) = -(M/2\pi\hbar^2)\langle\psi^{(-)}_{\vec{K}_f}(\vec{R})|H'_{fi}(\vec{R})|\psi^{(+)}_{\vec{K}_i}(\vec{R})\rangle \quad , \tag{2.7}$$

where, in our present time-independent formalism, $H'_{fi}(\vec{R})$ differs from the $H'_{fi}[\vec{R}(t)]$ of Eq. (2.2) only in the replacement of $H'[\vec{r},\vec{R}(t)]$ by the time-independent form $H'(\vec{r},\vec{R})$. As $R \to \infty$, the exact nuclear scattering wave function $\psi^{(+)}_{\vec{K}_i}$ behaves as

$$\psi^{(+)}_{\vec{K}_i}(\vec{R}) \sim e^{i\vec{K}_i\cdot\vec{R}} + F(\hat{R} \to \hat{K}_i)(e^{iK_iR}/R) \quad . \tag{2.8}$$

Similarly, since $\psi^{(-)*}_{\vec{K}} = \psi^{(+)}_{-\vec{K}}$, we have

$$\psi^{(-)*}_{\vec{K}_f}(\vec{R}) \sim e^{-i\vec{K}_f\cdot\vec{R}} + F(\hat{R} \to -\hat{K}_f)(e^{iK_fR}/R) \quad . \tag{2.9}$$

In line with approximation vii"), only the asymptotic forms of the nuclear wave functions are relevant. (Note that these forms contain the exact elastic scattering amplitudes.) The insertion of (2.8) and (2.9) into Eq. (2.7) gives four terms. We now make a peaking approximation:

ix") We drop all contributions involving exponentials which contain $\vec{K}_i$ or K_i and $\vec{K}_f$ or K_f unless the exponent can almost vanish, and we approximate any factor $g(\hat{K}_i,\hat{K}_f)$ of such an exponential by its

value for the direction(s) $\hat{K}_i$ and/or $\hat{K}_f$ at which the exponent almost vanishes. *We therefore drop the term proportional to exp* $i(\vec{K}_i-\vec{K}_f)\cdot\vec{R}$ since the exponent is negligible only for θ very small, a region we have excluded, *and we drop the term proportional to* R^{-2} *exp* $i(K_i+K_f)R$. In the coefficients of the terms with exp $i(\vec{K}_i\cdot\vec{R} + K_fR)$ and exp $i(-\vec{K}_f\cdot\vec{R} + K_iR)$, we approximate $\hat{R}$ by $-\hat{K}_i$ and by $\hat{K}_f$, respectively. Further, we use

$$K_i-K_f = (K_i^2-K_f^2)/(K_i + K_f) \approx (2M/\hbar^2)\hbar\omega/(2K_i) = \omega/v_i \quad .$$

Finally, since the angle of scattering is the same, we use rotational invariance to give $F(-\hat{K}_i \to -\hat{K}_f) = F(\hat{K}_i \to \hat{K}_f) = F_{el}(E_f,\theta)$, the last step following by definition, we drop terms in $\exp(\pm i)(K_i + K_f)R$ after having performed the angular integration over $d\vec{R}$, and we approximate $1/K_f$ by $1/K_i$ to arrive at

$$f(ion) = F_{el}(E_i,\theta)\int_0^\infty H'_{fi}(\hat{K}_fR)e^{i\omega R/v_i}\,dR/v_i + F_{el}(E_f,\theta)\int_0^\infty H'_{fi}(-\hat{K}_iR)e^{-i\omega R/v_i}\,dR/v_i \quad .$$

Setting $t = -R/v_i$ in the second integral and $t = R/v_i$ in the first, and introducing

$$Q \equiv F_{el}(E_f,\theta)/F_{el}(E_i,\theta) \quad , \tag{2.10}$$

we have

$$P(ion) = |f(ion)/F_{el}(E_i,\theta)|^2 = |I(0,\infty;\hat{K}_f) + QI(-\infty,0;\hat{K}_i)|^2/\hbar^2 \tag{2.11}$$

where the I's, defined by Eq. (2.4), are precisely those which appeared in the SCA.

We will be concerned with the case $\hbar\omega \ll E_i$, for the probability of the electron picking up an energy comparable to E_i is negligible. If we are off resonance, it follows that we can approximate K_f by K_i and therefore Q by 1, so that P(ion) is independent of $F_{el}(E_i,\theta)$, as is to be expected; with H' treated in first order perturbation theory, for a non-resonant nuclear reaction, the back reaction of the electron on P can be neglected, f(ion) is proportional

to P_{ef}, and P(ion) is independent of P_{ef}. For a resonant nuclear reaction, on the other hand, the slight change from E_i to E_f can be very important. In other words, in the time independent DWBA, with the further approximation of using the asymptotic forms of the nuclear scattering wave functions, the dependence of P(ion) on the half-life of the compound nucleus originates in the strong E dependence of F(E,θ) near a resonance. More precisely, on physical grounds we might expect the energy $\hbar\omega$ given up to the K electron to be of order U_K. (Formally, this follows from the quantum matrix element $H'_{fi}(\vec{R})$ for the electron defined by Eq. (2.2) with $H'(\vec{r},R(t))$ replaced by $H'(\vec{r},\vec{R})$. Thus, with a factor $\exp(-Z_T r/a_0)$ from ϕ_{1s} and in the approximation in which ϕ_f is proportional to $\exp(i\vec{k}_f\cdot\vec{r})$, we expect $H'_{fi}(\vec{R})$ to fall off rapidly for k_f larger than Z_T/a_0.) If F(E,θ) is to vary significantly as E varies by an amount U_K, one must have $\Gamma \lesssim U_K$.

C. Comparison of the SCA and the DWBA

Even though energy, momentum and angular momentum are conserved in the DWBA but not in the SCA, there is a close relation between the two approaches, and indeed we have already seen that they involve the identical I integrals. The analogy can be pursued further if $\omega\Delta t \ll 1$, and if we can approximate $F(E_f,\theta) = F(E_i-\hbar\omega,\theta)$ by $F(E,\theta) - \hbar\omega\partial F(E,\theta)/\partial E$, evaluated at $E = E_i$, so that, using the quantum mechanical time delay τ defined by

$$\tau = \tau(E,\theta) = -i\hbar\partial \ln F(E,\theta)/\partial E \quad ,$$

we can write $Q = 1-i\omega\tau$. In the SCA, we then have from Eqs. (2.5) and (2.3), writing the latter with an over-all factor $\exp(\frac{1}{2}i\omega\Delta t)$ and then using $\exp(-i\omega\Delta t) \approx 1-i\omega\Delta t$,

$$\hbar^2 P(\text{ion}) = \left|(1-i\omega\Delta t)I(-\infty,0;\hat{k}_i) + I(0,\infty;\hat{k}_f) + \Delta t H'_{fi}(0)\right|^2 . \quad (2.12)$$

To obtain P(ion) in the DWBA from Eq. (2.12) for P(ion) in the SCA, we must drop the last term and replace Δt by τ in the first term. The SCA and DWBA estimates of P(ion) may not be quite as close as they seem to be. Firstly, Δt is real while Δτ can be complex. Secondly, the DWBA P(ion) has no $H'_{fi}(0)$ term. Indeed, in the SCA, the $H'_{fi}(0)$ term originates in the $P-e^-$ interaction during the time P is at R = 0, while in the DWBA we used the asymptotic forms of $\psi_i^{(+)}$ and $\psi_f^{(-)}$, which are surely incorrect for small R and in particular at R = 0. To obtain the DWBA analog of the $H_{fi}(0)$ term, one must expand the nuclear wave functions (not their asymptotic forms) in partial waves, and study the monopole component of H', originating in the region R < r. The point is that the possibility of P penetrating the classically forbidden region is rather larger for the resonant than for the non-resonant case. Nevertheless, when all is said and done, the contribution from the region R < r will normally be quite small; the monopole contribution can be significant, but its inclusion does not change the form of Eq. (2.12), though a slight redefinition is necessary.

We close this section with a few comments on the literature.

Similar processes had been considered earlier, but the present process was first considered, in the SCA, by Ciochetti and Molinari (1965). Blair et al. (1978) recorded, without proof, the DWBA theoretical result. The analysis presented above follows very closely that of Feagin and Kocbach (1981). A proof of the DWBA result, somewhat of a tour de force, has been given by Blair and Anholt (1982). (The Appendix of this paper contains a study of various SCA analyses.) See also McVoy and Weidenmüller (1982). The first experiment showing the effect of the nuclear half-life on the ionization probability was described in the paper by Blair et al. referred to just above. In that experiment on ^{58}Ni and in a second on ^{86}Sr by Chemin et al. (1981), protons were elastically scattered across a resonance for which Γ was comparable in magnitude to U_K, and the effect was both expected and seen. In an experiment by Duinker et al. (1980) on the scattering of protons by ^{12}C across a resonance, the effect was seen even though it is not expected, since here Γ is much larger than U_K; this problem has not yet been resolved. A short but very nice review of a number of experiments which deeply involve the interplay of atomic and nuclear physics, not just the effect of a nuclear resonance on K-shell ionization considered here, has been given by Merzbacher (1982). See also "Two Notes on Sec. II" just before the list of references.

III. EFFECT OF COLLISIONS ON THE EMISSION OF RADIATION

A. Introduction

Consider a medium containing N two-level one-electron atoms. The n-th atom has the normalized wave function

$$\Psi^n(\vec{r}_n,\vec{R}_n,t) = A_1^n(\vec{R}_n,t)\psi_1(\vec{r}_n) + A_2^n(\vec{R}_n,t)\psi_2(\vec{r}_n) \quad , \quad (3.1)$$

where $\vec{R}_n$ locates the center of mass of the atom and where $\vec{r}_n$ is the electron coordinate relative to the atom's center of mass. The polarization (dipole moment density) of the medium at position $\vec{X}$ and time t is

$$\vec{P}(\vec{X},t) = N a\vec{r}_{12}\rho_{12}(\vec{X},t) + \text{c.c.} \quad , \quad (3.2)$$

where

$$\vec{r}_{12} = \int d\vec{r}\,\psi_1(\vec{r})\vec{r}\psi_2(\vec{r})^* \quad , \tag{3.3}$$

and where, assuming no correlations between atoms, ρ_{12} is an off-diagonal element of the ensemble average density matrix:

$$\rho_{ij}(\vec{X},t) = \frac{1}{N}\sum_{n=1}^{N} A_i^n(\vec{X},t)A_j^n(\vec{X},t)^* \quad . \tag{3.4}$$

The polarization of the medium or, equivalently, the "coherence" $\rho_{12}(\vec{X},t)$, governs the response of the medium to both an applied field and (as in spontaneous emission) to the vacuum field.

The population densities of states 1 and 2 in a macroscopically small region centered at $\vec{X}$ are $\rho_{11}(\vec{X},t)$ and $\rho_{22}(\vec{X},t)$, respectively. The total fractions of atoms in states 1 and 2 are $\int\rho_{11}(\vec{X},t)d\vec{X}$ and $\int\rho_{22}(\vec{X},t)d\vec{X}$. The populations change due to pumping and spontaneous emission (natural decay). Changes in populations due to collision induced transitions are neglected here. Changes in the coherence ρ_{12} occur because of pumping, natural decay, dipole dephasing between atoms of different velocities (Doppler broadening), and collisional decay (pressure broadening). We consider two-level atoms which do not interact with one another, and which can emit and absorb radiation and scatter from stationary (infinitely massive structureless) perturbers. Recently, new insight has been gained into the understanding of collisional effects on the coherence. We describe here the modern view of collisional effects on ρ_{12}, following closely the work of Berman (1975) and Berman et al. (1982).

B. Qualitative Discussion

Traditionally, the destruction of ρ_{12} by collisions is attributed to a loss of coherence of the phases for different values of n (phase interruption) of the products $A_1^n(\vec{R}_n,t)A_2^n(\vec{R}_n,t)^*$. (The traditional theory is described by, among others, Sobel'man (1972).) However, an alternative explanation, applicable in the classical regime, is that collisions destroy ρ_{12} by reducing the overlap of $A_1^n(\vec{R}_n,t)$ and $A_2^n(\vec{R}_n,t)^*$. To see this, we assume for simplicity that the collision is impulsive and that the scattering angle is small compared to unity. The effective potential between an active atom in state i and a perturber is, to a first approximation,

$$V_i(\vec{R}) = \int\psi_i(\vec{r})^* V(\vec{r},\vec{R})\psi_i(\vec{r})d\vec{r} \quad , \tag{3.5}$$

where $\vec{R}$ is the atom-perturber separation and $V(\vec{r},\vec{R})$ is their interaction. An atom entering a collision with momentum $\hbar\vec{k}$ in state i will scatter through an angle θ_i given approximately by

$$\theta_i = -\frac{1}{\hbar k}\int_{-\infty}^{\infty}\frac{\partial V_i(\vec{R})}{\partial b}\,dt \quad , \tag{3.6}$$

where in the integrand we may write $\vec{R} = \vec{b} + \vec{v}t$, with $\vec{b}$ and $\vec{v}$ the impact parameter and velocity of the incident atom. Assuming the diffraction angle 1/(kb) to be small, the scattering is classical if

$$\theta_i \gg 1/(kb) \quad . \tag{3.7}$$

If this condition is satisfied, the population $\rho_{ii}(\vec{R},t)$ follows a classical trajectory. The trajectories for the two populations are classically distinguishable if

$$|\theta_2 - \theta_1| \gg 1/(kb) \quad , \tag{3.8}$$

in which case the overlap of $A_1^n(\vec{R}_n,t)$ and $A_2^n(\vec{R}_n,t)$ is effectively zero after the collision. Therefore, assuming that the above inequalities hold, if an atom enters a collision in a superposition state, so that initially $\rho_{12} \neq 0$, the separation of population trajectories results in ρ_{12} vanishing after the collision. While this view differs from the traditional (phase interruption) view, the difference is not so great when one considers the manner in which the overlap $A_1^n(\vec{R}_n,t)A_2^n(\vec{R}_n,t)^*$ vanishes in the classical limit; the quantum-mechanical overlap acquires a large phase which varies rapidly with $\vec{R}_n$ and the overlap vanishes when averaged over slight variations of $\vec{R}_n$. It is interesting to combine Eqs. (3.6) and (3.8); approximating $\partial V_i/\partial b$ by V_i/b, Eq. (3.8) becomes

$$\hbar^{-1}\left|\int[V_2(R) - V_1(R)]dt\right| \gg 1 \quad . \tag{3.9}$$

The value of b for which the left-hand-side equals unity is denoted as b_w, the "Weisskopf radius". For $b < b_w$ collisions are classical and destroy ρ_{12} through trajectory separation (or phase interruption in the traditional view). For $b > b_w$ the scattering is nonclassical and does not destroy ρ_{12}. Thus ρ_{12} survives collisions only in the diffractive zone, that is, for $b > b_w$ and therefore in a narrow forward scattering cone.

C. Quantitative Discussion: Transport Equations

A more quantitative discussion requires the use of transport equations for the density matrix. To begin, we consider a beam of one-level atoms, each of mass m and incident velocity $\vec{v}$, scattering from a collection of perturbers with volume density N. The transport equation, in the form of Eq. (3.14b) below, could be written down without derivation simply on the basis of physical principles. However, we sketch a derivation here since it facilitates the derivation of the somewhat more complicated transport equations for two-level atoms. The atoms in the incident beam are assumed to have wavepackets of similar form but random impact parameters. Thus the normalized incoming wavepacket of a typical atom is

$$\phi_{in}(\vec{k}) = \exp(-i\vec{b}\cdot\vec{k})\phi_o(\vec{k})$$

where $\phi_o(\vec{k})$ is independent of the impact parameter $\vec{b}$ and is sharply peaked when $\vec{k}$ equals $m\vec{v}/\hbar$. After a collision, an atom is represented by the outgoing wavepacket $\phi_{out}(\vec{k})$, where (Taylor, 1972)

$$\phi_{out}(\vec{k}) = \phi_{in}(\vec{k}) + (1/\pi)\int d\vec{k}'\delta(k^2-k'^2)f(\vec{k}'\to\vec{k})\phi_{in}(\vec{k}'); \quad (3.10)$$

$f(\vec{k}'\to\vec{k})$ is the scattering amplitude. The ensemble average probability densities before and after the collision are $\rho_{in}(\vec{k})$ and $\rho_{out}(\vec{k})$, respectively, where the domain of integration is the cross sectional area A of the incident beam,

$$\rho_a(\vec{k}) = (1/A)\int d^2b|\phi_a(\vec{k})|^2 \quad , \quad [\text{i.e. } \rho_{in}(\vec{k}) = |\phi_o(\vec{k})|^2] \quad , \quad (3.11)$$

with a in or out. Now on average the beam encounters one perturber in the time interval $\tau = 1/(NvA)$. The collision rate of change of the probability density $\rho_{11}(\vec{k},t) \equiv \rho(\vec{k},t)$ is therefore

$$\left.\frac{\partial\rho(\vec{k},t)}{\partial t}\right|_{coll} = \frac{\rho_{out}(\vec{k})-\rho_{in}(\vec{k})}{\tau}$$

$$= Nv\int d^2b\,[|\phi_{out}(\vec{k})|^2 - |\phi_{in}(\vec{k})|^2] \quad . \quad (3.12)$$

If Eq. (3.10) is used to substitute for $\phi_{out}(\vec{k})$ in Eq. (3.12), the integration over b may be done by assuming A to be sufficiently large that we can use, for $\vec{k} \neq \vec{k}'$

$$\int d^2b\phi^*_{in}(\vec{k}')\phi_{in}(\vec{k}) = (2\pi)^2\delta^2(\vec{k}'_\perp - \vec{k}_\perp)\phi^*_o(\vec{k}')\phi_o(\vec{k}) \quad , \quad (3.13)$$

where $\vec{k}_\perp$ denotes the component of $\vec{k}$ perpendicular to $\vec{v}$. An integration over the variable $\vec{k}'$ may then be done using the fact that $\phi_o(\vec{k}')$ is highly localized --- see the analogous discussion of Taylor (1972), pages 49-51. Setting $\rho_{in}(\vec{k}) = \rho(\vec{k},t)$, the following transport equation is obtained:

$$\left.\frac{\partial\rho(\vec{k},t)}{\partial t}\right|_{coll} = -\Gamma(\vec{k})\rho(\vec{k},t) + \int d\vec{k}'W(\vec{k}'\to\vec{k})\rho(\vec{k}',t) \quad , \quad (3.14a)$$

where, with $\sigma(\vec{k})$ denoting the total cross section,

$$W(\vec{k}'\to\vec{k}) = Nv|f(\vec{k}'\to\vec{k})|^2\, k^{-2}\delta(k-k') \quad , \quad (3.15)$$

$$\Gamma(\vec{k}) = 4\pi(N\hbar/m)\mathrm{Im}f(\vec{k}\to\vec{k}) = Nv\sigma(\vec{k}) = \int d\vec{k}'W(\vec{k}\to\vec{k}') \quad . \quad (3.16)$$

Note that $k = k'$ in Eq. (3.14a) since the perturbers do not recoil. Further, $k = mv/\hbar$ since $\phi_o(\vec{k})$ is highly localized. However, since Eq. (3.14a) is linear, it applies when $\rho_{in}(\vec{k})$ is a superposition of densities localized in *different* regions of $\vec{k}$-space; hence $\rho(\vec{k},t)$ may represent a broad distribution in $\vec{k}$-space. From Eqs. (3.15) and (3.16), Eq. (3.14a) can be written in the more transparent form

$$\left.\frac{\partial\rho(\vec{k},t)}{\partial t}\right|_{coll} = -Nv\sigma(\vec{k})\rho(\vec{k},t)+Nv\int d\hat{k}'|f(\vec{k}'\to\vec{k})|^2\rho(\vec{k}',t). \quad (3.14b)$$

The transport equations for the density matrix $\rho_{ij}(\vec{k},t)$, with $\rho_{ij}(\vec{k},t)$ the momentum space analog of $\rho_{ij}(\vec{k},t)$ of Eq. (3.4), of two-level atoms can be derived similarly if collision induced transitions between the two levels are neglected. We have (Berman, 1975)

$$\left.\frac{\partial\rho_{11}(\vec{k},t)}{\partial t}\right|_{coll} = -\Gamma^{vc}_{11}(\vec{k})\rho_{11}(\vec{k},t) + \int d\vec{k}'W_{11}(\vec{k}'\to\vec{k})\rho_{11}(\vec{k}',t), \quad (3.17a)$$

$$\left.\frac{\partial\rho_{12}(\vec{k},t)}{\partial t}\right|_{coll} = -(\Gamma^{ph}_{12}(\vec{k}) + \Gamma^{vc}_{12}(\vec{k}))\rho_{12}(\vec{k},t)$$

$$+ \int d\vec{k}'W_{12}(\vec{k}'\to\vec{k})\rho_{12}(\vec{k}',t) \quad , \quad (3.17b)$$

where, if $f_i(\vec{k}\to\vec{k}')$ is the scattering amplitude for an atom in state i,

$$W_{ij}(\vec{k}'\to\vec{k}) = Nvf_i(\vec{k}'\to\vec{k})f^*_j(\vec{k}'\to\vec{k})k^{-2}\delta(k-k') \quad . \quad (3.18)$$

$$\Gamma_{ij}^{vc}(\vec{k}) = \int d\vec{k}' W_{ij}(\vec{k} \to \vec{k}') \quad . \tag{3.19}$$

$$\Gamma_{12}^{ph}(\vec{k}) = -2\pi i N(\hbar/m)[f_1(\vec{k} \to \vec{k}) - f_2^*(\vec{k} \to \vec{k})] - \Gamma_{12}^{vc}(\vec{k}) \quad . \tag{3.20}$$

The superscript vc denotes velocity changing. If velocity changing collisions are neglected, that is, if $W_{ij}(\vec{k} \to \vec{k}') = \bar{W}_{ij}(\vec{k})\delta^3(\vec{k}'-\vec{k})$ so that $\Gamma_{ij}^{vc}(\vec{k}) = \bar{W}_{ij}(\vec{k})$, the integral term and the term in Γ_{ij}^{vc} cancel in Eqs. (3.17), and these equations reduce to the much simpler equations of the traditional pressure broadening theory:

$$\left.\frac{\partial \rho_{ii}(\vec{k},t)}{\partial t}\right|_{coll} = 0 \quad . \tag{3.21a}$$

$$\left.\frac{\partial \rho_{12}(\vec{k},t)}{\partial t}\right|_{coll} = -\Gamma_{12}^{ph}(\vec{k})\rho_{12}(\vec{k},t) \quad . \tag{3.21b}$$

With velocity changing collisions neglected, Γ_{12}^{ph} is, in the traditional view, the collision decay rate due to phase interruption of the atomic dipole. If velocity changing collisions are allowed but if the scattering amplitudes f_1 and f_2 are equal, it follows from Eqs. (3.18)-(3.20) and the optical theorem that $\Gamma_{12}^{ph}(\vec{k}) = 0$; thus there is no phase interruption of the atomic dipole during the collision if the atom scatters as a structureless entity.

The qualitative analysis of the previous subsection indicates that, in general, $W_{12}(\vec{k} \to \vec{k}')$ vanishes in the classical scattering regime due to trajectory separation. (More accurately, W_{12} oscillates rapidly and the integral over the classical region vanishes.) W_{12} gives a nonvanishing contribution only in the diffractive scattering region. Thus any departure from a transport equation for ρ_{12} of the form of Eq. (3.21b) arises from diffractive scattering. Such a departure may be observed by creating a photon echo, as we now briefly discuss.

D. Laser Spectroscopy

Suppose that a single-mode laser of frequency Ω interacts with the atoms. Assume the laser field $\vec{E}$ propagates in the z direction, i.e., $\vec{E} = \vec{E}_0\cos(Kz-\Omega t)$. In the absence of collisions and under appropriate initial conditions $\rho_{12}(\vec{k},t)$ factors into

$$\rho_{12}(\vec{k},t) = \rho_{12}(\vec{k}_T)\rho_{12}(k_z,t) \tag{3.22}$$

where, following convention, we use $\rho_{12}(x)$ to denote different

functions for different arguments x, and where $\vec{k}_T$ is the component of $\vec{k}$ perpendicular to the z-axis. This factorization is assumed to hold, to a first approximation, in the presence of collisions. Substituting into Eq. (3.17b), integrating over $\vec{k}_T$, and defining the normalization of $\rho_{12}(\vec{k}_T)$ --- it is not defined by Eq. (3.22) --- to be $\int \rho_{12}(\vec{k}_T)d^2k_T = 1$, we find

$$\left.\frac{\partial \rho_{12}(k_z,t)}{\partial t}\right|_{coll} = -[\Gamma_{12}^{vc}(k_z) + \Gamma_{12}^{ph}(k_z)]\rho_{12}(k_z,t) + \int_{-\infty}^{\infty} dk_z' W_{12}(k_z' \to k_z)\rho_{12}(k_z',t) \quad , \tag{3.23}$$

$$W_{12}(k_z' \to k_z) = \int d^2k_T \int d^2k_T' W_{12}(\vec{k}' \to \vec{k})\rho_{12}(\vec{k}_T') \quad , \tag{3.24}$$

$$\Gamma_{12}^{\beta}(k_z) = \int d^2k_T \Gamma_{12}^{\beta}(\vec{k})\rho_{12}(\vec{k}_T) \quad , \tag{3.25}$$

where β is vc or ph.

The field frequency seen in the rest frame of an atom is $\tilde{\Omega} = \Omega - Kv_z$ (Doppler shift) where $K = \Omega/c$ and $v_z = \hbar k_z/m$. If $|\tilde{\Omega} - \omega| << |\tilde{\Omega} + \omega|$, where ω is the transition frequency for the two atomic levels, $\rho_{12}(k_z,t)$ will oscillate in time with the field as $\rho_{12}(k_z,t) = \tilde{\rho}_{12}(k_z,t)\exp(i\tilde{\Omega}t)$ where $\tilde{\rho}_{12}(k_z,t)$ varies slowly with time (rotating wave approximation). From Eq. (3.23), we then have (Berman et al., 1982)

$$\left.\frac{\partial \tilde{\rho}_{12}(k_z,t)}{\partial t}\right|_{coll} = -[\Gamma_{12}^{vc}(k_z) + \Gamma_{12}^{ph}(k_z)]\tilde{\rho}_{12}(k_z,t) + \int_{-\infty}^{\infty} dk_z' W_{12}(k_z' \to k_z)\exp[iK(v_z - v_z')t]\tilde{\rho}_{12}(k_z',t) \quad . \tag{3.26}$$

(Note that since we are concerned here with collisional effects, we have not included a term originating in $(\partial/\partial t)\exp(i\tilde{\Omega}t)$.) This equation governs the collision rate of change of ρ_{12} in the presence of a single-mode laser. To obtain the full rate of change of ρ_{12} with time, the impact approximation is assumed. In this approximation a collision is regarded as instantaneous compared to all other relevant time scales. Then $\partial\rho_{12}/\partial t|_{coll}$ can be simply added to the time derivative $\partial\rho_{12}/\partial t|_{rad}$ due to coupling with the radiation field to give the full time derivative.

Only the diffractive scattering contributes to the integral of

Eq. (3.26). Now diffractive scattering occurs in a very narrow forward cone (assuming the atoms are not moving too slowly) and so the accompanying velocity changes are small. Let δv be the characteristic value of the velocity change $v_z' - v_z$ in the diffractive region. A coherence $\tilde{\rho}_{12}(k_z,0)$ which is prepared at time $t = 0$ will subsequently decay; let τ_c be the coherence lifetime. If $K\delta v\tau_c << 1$ the exponential in the integrand of Eq. (3.26) may be set equal to unity. Further, $\tilde{\rho}_{12}(k_z',t)$ is expected to vary slowly over the diffractive region so that it may be taken out of the integral of Eq. (3.26) at the value $\tilde{\rho}_{12}(k_z,t)$. In this case, the term in Γ_{12}^{vc} cancels the integral term and Eq. (3.26) reduces to the traditional equation

$$\left.\frac{\partial\tilde{\rho}_{12}(k_z,t)}{\partial t}\right|_{coll} = -\Gamma_{12}^{ph}(k_z)\tilde{\rho}_{12}(k_z,t) \quad , \tag{3.27}$$

leading to the prediction that $\tilde{\rho}_{12}$ has the decay rate $\mathrm{Re}\Gamma_{12}^{ph}(k_z)+\gamma_{12}$, where γ_{12} is the natural decay rate. (If the experimental situation involves a distribution of k_z, we must average over k_z and include the free induction decay rate due to a relative dephasing of atomic dipoles with different velocities.) However, suppose instead that $K\delta v\tau_c >> 1$. For $t > 1/(K\delta v)$ the exponential oscillates rapidly over the diffractive region and the integral in Eq. (3.26) vanishes so we obtain (Berman et al., 1982)

$$\left.\frac{\partial\tilde{\rho}_{12}(k_z,t)}{\partial t}\right|_{coll} = -[\Gamma_{12}^{vc}(k_z) + \Gamma_{12}^{ph}(k_z)]\tilde{\rho}_{12}(k_z,t) \quad , \tag{3.28}$$

leading to the prediction of the larger decay rate $\mathrm{Re}[\Gamma_{12}^{vc}(k_z) + \Gamma_{12}^{ph}(k_z)] + \gamma_{12}$. Now τ_c, the effective coherence lifetime of ρ_{12}, depends on the experimental situation. Without going into the details of a photon echo --- a lucid discussion is given in Sargent et al., 1974 --- suffice it to say that in a photon echo experiment τ_c can be made large; the condition $K\delta v\tau_c >> 1$ can therefore be attained and the larger decay rate confirmed. This was done recently (Mossberg et al., 1980), establishing for the first time the influence of diffractive scattering on the emission of radiation.

IV. ASYMMETRIC CHARGE TRANSFER

A measure of the great practical importance of the charge transfer process is the very considerable experimental and theoretical effort devoted to that process. The areas in which significant progress has very recently been recorded include atom capture as well as electron capture, the eikonal approximation, and versions of the Glauber approximation. Unfortunately, space permits only one topic, an important step in our understanding of asymmetric charge transfer.

To appreciate this step, it will be useful to consider earlier developments in asymmetric and symmetric charge transfer. There have been a number of relatively recent reviews in these areas (Basu et al., 1978; Belkić et al., 1979; Shakeshaft and Spruch, 1979; Shakeshaft, 1982), and we limit ourselves to some brief comments.

We wish to consider electron capture by a projectile P (a bare nucleus of charge $Z_p e$) incident with a high velocity $\vec{v}$ on a neutral atom. The target nucleus T has a charge $Z_t e$, and the process is

$$P^{(Z_p)} + \mathrm{Atom} \longrightarrow P^{(Z_p-1)} + \mathrm{Ion}^{(+1)} \quad .$$

One must and can do better (Briggs and Taulbjerg, 1979) but we assume that all electron-electron interactions are negligible. In a Born expansion in a non-relativistic context, the n-th term represents the contribution associated with n scatterings. We make one remark on potential scattering (scattering by a target with no internal degrees of freedom) before considering charge transfer. The Born expansion, for sufficiently large incident energy E and for many potentials, is a convergent expansion in powers of $\bar{V}/E$, where $\bar{V}$ is a characteristic value of V. The first Born term therefore dominates for sufficiently large E for potential scattering.

We begin our consideration of charge transfer by studying the symmetric case, for which $Z_p = Z_T \equiv Z$. In the first Born term, the main contribution originates in components of the target and final bound state wave functions for which the velocity of the electron is comparable with $\vec{v}$, and those amplitudes are very small for v large --- more precisely, for $v >> Ze^2/\hbar$, a characteristic electron velocity in the initial and final state --- even if, as we do for simplicity, we consider capture from and to ground states. The second Born term can be described roughly as follows. The electron can initially have a small speed, for it is given a speed close to v in a close collision with P. The electron then moves, in this intermediate state, almost as a free particle. (The uncertainty in its energy is $\Delta E \sim p\Delta p/m \sim \hbar v/a$, with a an atomic dimension, so that $\Delta E/E$ falls off as $1/v$, but the off-the-energy shell component gives a significant contribution). The electron is then scattered elastically by T, emerging with velocity close to $\vec{v}$, and is captured. The second Born term dominates over the first even though it involves an additional collision (and therefore an additional factor, proportional to e^2, which often suggests that the term involved is of higher order) because the second Born term does not require high speed components in the initial and final bound states. It is widely believed, though it has not been proved, that higher order Born terms are dominated by the second, for they suffer from having still further factors of e^2, and they have no compensating advantages since the second Born term already allows low velocities in

the initial and final states.

For many applications of great current interest, one has Z_p small, say unity, but $Z_T \geq 5$, and incident energies E such that v is rather large compared to $Z_p e^2/\hbar$ but not compared to $Z_T e^2/\hbar$. It is then inappropriate to ignore multiple e^--T collisions; rather, all e^--T collisions must be included. However, we can continue to ignore multiple e^--P collisions. In the present asymmetric analog of the second Born term in the symmetric case, the electron in the intermediate state is described by a Coulomb wave rather than by a plane wave. A natural starting point is to assume that the Coulomb wave is on the energy shell. This amounts to the impulse approximation, developed largely in this context by Briggs (1977) and also by Kocbach (1980) and Amundsen and Jakubassa (1980). While this approach gives good results at larger incident energies, theory and experiment begin to disagree at energies rather above the value at which the disagreement had been expected. The point is that the off-the-energy-shell component of the intermediate state wave function --- now a Coulomb wave --- must be retained. The analysis is tricky, and requires further approximations. It is a major achievement that *the final result is obtained in tractable form: the predicted asymmetric* charge transfer cross section differs from the impulse approximation prediction by a rather simple factor, one which gives considerably better agreement with the data at lower energies. We note incidentally that this work not only provides a theoretical foundation for asymmetric charge transfer but also provides much deeper insight into a number of earlier approaches, placing them in a hierarchy of successive approximations. See Macek and Taulbjerg (1981), Briggs, Macek and Taulbjerg, to be published, and Macek and Alston, to be published.

Two Notes on Sec. II:

1) Many intermediate nuclear half-lives can be determined by means independent of the measurement of P(ion) --- most generally by matching scattering data to the Breit-Wigner formula, but also by using special techniques, such as channeling. One time interval which might be determined most easily by a measurement of P(ion) is the time interval during which two heavy ions remain in one another's neighborhood in the course of a scattering process.

2) The argument of the first paragraph of Sec. II can be reversed; one can use a detailed knowledge of the properties of a nuclear resonance to determine an atomic property. Thus, let us rewrite the equation above Eq. (2.10) as

$$f(\text{ion}) = F_{ef}(E_i,\theta)A + F_{ef}(E_f,\theta)B$$

where A and B depend only upon atomic properties. This form remains valid even if one includes contributions from small values of R. Now assume, for example, that the nuclear resonant state is an $s_{\frac{1}{2}}$ state, and choose θ to be 90° so that the only relevant interference term arises from the monopole term, with both P and e^- emerging in spherically symmetric distributions. One can then show that $B = A^*$. If one were not at a resonance, one would have $F_{ef}(E_i,\theta) = F_{ef}(E_f,\theta)$, and therefore $f(\text{ion}) = 2\,F_{ef}(E_i,\theta)\text{Re}A$. At resonance, however, one can also, at least in principle, determine the imaginary component of A. It is not clear however if theory and experiment are now good enough to determine ImA. (See Blair et al., 1978, and references therein.)

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